**Part B)**

Over the past two weeks our team has worked to expand the scope of our design space now that we have developed our search algorithms and better understand the general tradeoffs between the independent variables. To this end, we have added the following aspects to the design space:

* Two additional levels to the “surface power” independent variable to represents mixed nuclear/solar and mixed nuclear/fuel cell technologies
* Seven additional levels to the “location” independent variable to represent other potential landing sites
* The new independent variable “transit fuel” representing the percentages of fuel provided from Earth or Lunar ISRU
* The new independent variable “return fuel” representing the percentages of fuel provided from Earth or Martian ISRU
* The new independent variable “entry type” representing two different atmospheric entry and deceleration technologies
* The new independent variable “staging location” representing three possible near earth orbit locations for mission staging.

The table below displays the full set of independent variables and all levels of these variables. The color coding represents the results of sensitivity studies where the darker green represents “better” solutions that maximize the objective of science value per dollar, while lighter areas and white areas represent “worse” solutions around an initial guess (not ultimately found to be X\*).

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Propulsion** | **Surface Power** | **Location** | **Food** | **Surface Crew** | **Transit Fuel** | **Return Fuel** | **Entry Type** | **Staging Location** |
| LH2 - 445 | Nuclear | Gusev | 100% Earth | 24 | All Earth | All Earth | Aerocapture | LEO |
| LH2 - 452 | Solar | Holden | 75%E, 25%M | 18 | Earth LH2; Lunar O2 | Earth LH2; Mars O2 | Propulsive | EML1 |
| LH2 - 465 | Nuclear/Solar | Eberswalde | 50-50 split | 12 | All Lunar | All Mars |  | EML2 |
| LH2 - 480 | Nuclear/FuelCell | Gale | 25%E, 75%M |  |  |  |  |  |
| NTR - 850 |  | Mawrth | 100% Mars |  |  |  |  |  |
| NTR - 950 |  | Utopia |  |  |  |  |  |  |
| NTR - 1000 |  | Hellas |  |  |  |  |  |  |
|  |  | Meridiani |  |  |  |  |  |  |
|  |  | Planus Boreum |  |  |  |  |  |  |
|  |  | Elysium |  |  |  |  |  |  |
|  |  | Amazonis |  |  |  |  |  |  |
|  |  | Isidis |  |  |  |  |  |  |

The new size of the design space is 272,160 potential architectures. With an average model run time of 0.8 seconds per design, a full factorial enumeration of the design space would equate to roughly 60.5 hours of computation time. Therefore, the design space is beyond the capability of this research group to fully explore and the coordinate-based search algorithm and genetic algorithm developed previously shall be used to determine the optimal Mars2040 mission architecture and investment portfolio.

In order to complete section (b1), the coordinate-based search analysis conducted in assignment 3 was re-run for the expanded design space to find a new optimal point, X\*. The table below displays the results from the coordinate search.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Starting Point** | **Optimal Solution** | **Value** | **Iterations** | **Function Evals** | |
| [1,0,0,0,0,2,2,0,2] | [2,0,0,0,0,2,2,0,2] | 1.04E-04 | 4 | 31 | |
| [3,1,2,2,1,0,0,0,0] | [0,2,0,0,0,2,0,0,0] | 1.23E-04 | 14 | 166 | |
| [6,3,11,4,2,0,0,1,0] | [4,3,0,0,0,0,2,0,0] | 9.96E-05 | 25 | 259 | |
| [0,0,0,0,0,0,0,0,0] | [0,2,0,0,0,2,0,0,0] | 1.23E-04 | 7 | 59 | |
| [1,1,1,1,1,1,1,1,1] | [0,2,0,0,0,2,2,0,0] | 2.05E-04 | 12 | 140 | |
| [2,2,2,2,2,2,2,0,2] | [2,2,0,0,0,2,2,0,2] | 1.04E-04 | 9 | 101 | |
| [3,3,3,3,1,1,1,1,1] | [2,3,0,0,0,2,2,0,2] | 1.04E-04 | 15 | 178 | |
| [4,1,7,4,1,1,1,1,1] | [0,2,0,0,0,2,2,0,0] | 2.05E-04 | 24 | 286 | |
| [5,2,10,2,1,1,1,1,1] | [0,2,0,0,0,2,2,0,0] | 2.05E-04 | 25 | 294 | |
| [0,2,0,0,0,2,2,0,0] | [0,2,0,0,0,2,2,0,0] | 2.05E-04 | 3 | 21 | |
| Global Maximum | | Local Maximums (darker is worse) | | |

It should be noted that the larger design space presents a variety of local maximum values, but with the coordinate search started at multiple points around the design space it rather efficiently finds what appears to be the global minimum. The total computational requirements of the coordinate search with 10 initial starting points spread through the design space was 1535 function evaluations. The global maximum found corresponds to the following optima architecture, X\*:

* LH2 propulsion
* 445 seconds ISP
* Nuclear/Solar surface power
* Gusev landing site
* 100% earth supplied food
* 24 person surface crew
* Lunar LH2 and Lunar O2 ISRU for transit fuel
* Mars LH2 and Mars O2 ISRU for return fuel
* Aerocapture entry type
* LEO staging location

The remainder of (b1) shall be conducted using this new X\* for our expanded design space.

**(b1) Scaling**

(b1.1) We computed the 9 diagonal entries of the Hessian at our current optimal solution found using the gradient-free local search algorithm described above. Central finite differencing was used except for cases where the optimal design vector was on a constraint. In those cases we used either forward or backward differencing in order to evaluate neighboring feasible points. Also, for variables with only two discrete options (eg Entry Type) we set the Hessian to zero. The diagonal entries for X\* are:

H(1,1)= -0.019\*10-3 Propulsion (ISP and type)

H(2,2)= -0.001\*10-3 Surface power

H(3,3)= 0.034\*10-3 Location

H(4,4)= -0.0002\*10-3 Food

H(5,5)= -0.008\*10-3 Crew

H(6,6)= -0.0542\*10-3 Transit Fuel Source

H(7,7)= -0.083\*10-3 Return Fuel Source

H(8,8)= 0 Entry Type

H(9,9)= -.0127\*10-3 Staging Location

(b1.2) Eight of the nine diagonal entries of the Hessian are less than 10-2, so we must scale our objective function which is currently Science Utility per million dollars. We chose to scale this objective by multiplying it by 105. This makes our new objective units Science Utility per $10. The result of this scaling is that the diagonal values of our Hessian are all between magnitude 10-2 and 102 with a resulting condition number of 2.33. This condition number is near a value of one suggesting our problem is now well scaled.

(b1.3) Scaling the objective itself did not change the optimal design point, meaning that the poor scaling may have not been affecting the search behavior. This is likely because the Hessian values (or the objective function itself) are not near Matlab’s double precision or machine epsilon of 2.2204e-16.

H(1,1)= -1.88 Propulsion (ISP and type)

H(2,2)= -1.10 Surface power

H(3,3)= 3.42 Location

H(4,4)= -0.02 Food

H(5,5)= -0.773 Crew

H(6,6)= -5.42 Transit Fuel Source

H(7,7)= -8.29 Return Fuel Source

H(8,8)= 0 Entry Type

H(9,9)= -12.67 Staging Location

The table below displays the coordinate-search results for the scaled problem. It can easily be seen that the search algorithm used the same computational power and came to find the same solutions, abliet the “value” metric is now appropriately scaled.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Starting Point** | **Optimal Solution** | **Value** | **Iterations** | **Function Evals** | |
| [1,0,0,0,0,2,2,0,2] | [2,0,0,0,0,2,2,0,2] | 10.40 | 4 | 31 | |
| [3,1,2,2,1,0,0,0,0] | [0,2,0,0,0,2,0,0,0] | 12.31 | 14 | 166 | |
| [6,3,11,4,2,0,0,1,0] | [4,3,0,0,0,0,2,0,0] | 9.96 | 25 | 259 | |
| [0,0,0,0,0,0,0,0,0] | [0,2,0,0,0,2,0,0,0] | 12.31 | 7 | 59 | |
| [1,1,1,1,1,1,1,1,1] | [0,2,0,0,0,2,2,0,0] | 20.51 | 12 | 140 | |
| [2,2,2,2,2,2,2,0,2] | [2,2,0,0,0,2,2,0,2] | 10.45 | 9 | 101 | |
| [3,3,3,3,1,1,1,1,1] | [2,3,0,0,0,2,2,0,2] | 10.45 | 15 | 178 | |
| [4,1,7,4,1,1,1,1,1] | [0,2,0,0,0,2,2,0,0] | 20.51 | 24 | 286 | |
| [5,2,10,2,1,1,1,1,1] | [0,2,0,0,0,2,2,0,0] | 20.51 | 25 | 294 | |
| [0,2,0,0,0,2,2,0,0] | [0,2,0,0,0,2,2,0,0] | 20.51 | 3 | 21 | |
| Global Maximum | | Local Maximums (darker is worse) | | |

**(b2) Multiobjective Optimization**

(b2.1)

(b2.2)

(b2.3)

(b2.4)